Identifying System-Wide Early Warning Signs of Instability in Stochastic Power Systems

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***Abstract*—Prior research has shown that spectral decomposition of the reduced power flow Jacobian (RPFJ) can yield participation factors that describe the extent to which particular buses contribute to particular spectral components of a power system. Research has also shown that both variance and autocorrelation of time series voltage data tend to increase as a power system with stochastically fluctuating loads approaches certain critical transitions. This paper presents evidence suggesting that a system's participation factors predict the relative bus voltage variance values for all nodes in a system. As a result, these participation factors can be used to filter, weight, and combine real time PMU data from various locations dispersed throughout a power network in order to develop coherent measures of global voltage stability. This paper first describes the method of computing the participation factors. Next, two potential uses of the participation factors are given: (1) predicting the relative bus voltage variance magnitudes, and (2) locating generators at which the autocorrelation of voltage measurements clearly indicate proximity to critical transitions. The methods are tested using both analytical and numerical results from a dynamic model of a 2383-bus test case.**

***Index Terms*—Power system stability, phasor measurement units, time series analysis, autocorrelation, critical slowing down, spectral analysis.**

I. Introduction

On a particularly hot day in July of 1987, the power system infrastructure in Tokyo Japan saw a dramatic increase in demand as millions of air conditioning units were turned on line. This demand spike occurred very rapidly and caused system wide voltages to sag. Voltage collapse soon followed, leaving almost 3 million people without electrical power. According to [1] and [2], inadequate operational planning coupled with poor situational awareness were primary causes of the blackout. Unfortunately, this is not an isolated voltage collapse incident: in order to optimize limited infrastructure, many power systems are frequently operated close to critical (or bifurcation) points, leaving them vulnerable to the devastating effects of voltage collapse. This paper seeks to improve situational awareness by providing a method that can use information gleaned from network models to combine streams of synchrophasor data in ways that provide useful information about a particular system's proximity to stability limits. Ultimately, the goal of this work is to provide operators with enhanced tools for gauging a system's long-term stability. In this paper, we particularly focus on identifying early warning signs of *long term voltage stability*, as defined by a joint IEEE PES/CIGRE task force [3].

There is increasing evidence that as a dynamical system approaches a bifurcation, early warning signs (EWSs) of the looming transition appear in the statistical properties of time series data from that system. This fact has been evidenced in many complex systems, including ecological networks, financial markets, the human brain, and power systems [4], [5]. Researchers have even found that human depression onset can be predicted by these same statistical properties [6]. In the statistical physics literature this phenomenon is known as Critical Slowing Down (CSD) [7]. When stressed, systems experiencing CSD require longer periods to recover from stochastic perturbations. Two of the most well-documented signs of CSD are increased variance and autocorrelation [5].

Real power systems, particularly as renewable energy production increases, are constantly subject to stochastically fluctuating supply and demand. The presence of stochastic power injections has motivated research to quantify the presence of CSD in bulk power networks, particularly as early warning signs of bifurcations such as voltage collapse (a type of Saddle-node bifurcation [8]) or oscillatory instability (a type of Hopf bifurcation). Through simulations, reference [9] demonstrated that both variance and autocorrelation in bus voltages increase substantially as several power systems approached saddle node bifurcation. Similarly, reference [10] computes an auto-correlation function for a power system model to gauge collapse probability. Finally, variance and autocorrelation are measured in an unstable power system in [4] across many state variables. These results indicate that variance of bus voltages and autocorrelation of line currents show the most useful signals of CSD, whereas current angles, voltage angles, generator rotor angles, and generator speeds did not generally yield sufficiently strong signs of CSD to provide actionable early warning of bifurcation.

Although CSD does consistently appear in these systems before bifurcations, not all variables in a complex system exhibit CSD sufficiently early to be useful EWSs [11]. For instance, reference [4] destabilized a simulated power system by over stressing all load buses. Signals were then collected from many nodes in this system, and certain nodes did not conclusively show measurable early CSD warning signs. In order to mitigate this problem, we propose a method that combines measurements from a variety of locations using spectral analysis of the power flow Jacobian, as introduced in reference [12]. By understanding which variables are the best

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indicators of long-term voltage stability, we aim to develop measures that are useful for assessing the stability of an entire system.

It is well known that voltage collapse can cause non-convergence in AC power flow solvers[8]. Additionally, reference [13] shows that only under very strict conditions will the load flow Jacobian show unambiguous signs of dynamic instability. Since our experiments do not meet these conditions, spectral analysis of the load flow Jacobian, as proposed in this paper, does not provide direct warning signs of dynamic instability. However, since spectral analysis is used in our approach merely as a means of deciding how to combine PMU data from diverse locations, our method provides a complement to conventional approaches that focus only on spectral analysis of the load flow Jacobian. It is also well known that the saddle-node bifurcation is the maximum upper limit on system loadability. But this upper limit is seldom reached, because system dynamics generally become unstable well before the saddle-node bifurcation occurs [13]. Indeed, reference [14] shows that a Hopf bifurcation will frequently precede a saddle-node bifurcation. Therefore, monitoring for voltage instability is only one important aspect of overall power system stability;there is a strong link between the occurrence of a Hopf bifurcation and the proximity of a saddle-node bifurcation.

By combining spectral analysis and CSD theory, this paper shows that the RPFJ contains valuable information about voltage stability. We show that participation factors resulting from a spectral analysis of this matrix can be used to weight and filter real-time PMU data, thus suggesting a method for combining the data into low-dimensional metrics of long-term voltage stability. This paper does not seek to define these metrics in detail; instead, we present a tool (spectral analysis of the RPFJ) that provides a foundation for the development of such metrics in future work. Section II of this paper outlines the mathematical methods and motivation for forming and decomposing the RPFJ. Section III presents the 2383 test case and shows the potential usefulness of the participation factors Finally, our conclusions are presented in Section IV.

II. Spectral Analysis of the Power Flow Jacobian

This section presents a method for using spectral decomposition of the RPFJ to identify and weight variables that will most clearly show evidence of CSD. Further information on this spectral decomposition approach can be found in [12].

The standard power flow Jacobian matrix, which is a linearization of the steady state power flow equations, is given by (1).

$$\begin{bmatrix} \Delta \mathbf{P} \\ \Delta \mathbf{Q} \end{bmatrix} = \begin{bmatrix} J_{\mathbf{P}\boldsymbol{\theta}} & J_{\mathbf{PV}} \\ J_{\mathbf{Q}\boldsymbol{\theta}} & J_{\mathbf{QV}} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\theta} \\ \Delta \mathbf{V} \end{bmatrix} \tag{1}$$

In order to perform V-Q sensitivity analysis (an important aspect of voltage stability analysis), we assume that the incremental change in real power $\Delta \mathbf{P}$ is equal to 0. In this way, we can study how incremental changes in injected reactive power affect system voltages. Setting $\Delta \mathbf{P} = \mathbf{0}$ and rearranging terms to remove $\Delta \boldsymbol{\theta}$, the expression for the reduced Jacobian is defined:

$$\Delta \mathbf{Q} = \left[J_{\mathbf{QV}} - J_{\mathbf{Q}\boldsymbol{\theta}} J_{\mathbf{P}\boldsymbol{\theta}}^{-1} J_{\mathbf{PV}} \right] \Delta \mathbf{V} = [J_R]\, \Delta \mathbf{V} \tag{2}$$

Assuming that Newton-Raphson converges to a power flow solution for the system being studied, the matrix J_R is non-singular and can be written as the product of its right eigenvector matrix R, its left eigenvector matrix L, and its diagonal eigenvalue matrix Λ, such that:

$$J_R = R \Lambda L \tag{3}$$

The left and right eigenvectors can then be orthonormalized such that, for the right eigenvector $\mathbf{r}_i$ (column vector) and the left eigenvector $\mathbf{l}_j$ (row vector), the Konecker delta function defines their relationship:

$$\mathbf{l}_j \mathbf{r}_i = \delta_{j,i} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \tag{4}$$

We begin our method by decomposing J_R using a simple similarity transform. The transform is substituted into (2):

$$\begin{aligned} \Delta \mathbf{Q} &= \\ & \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \cdots & \mathbf{r}_n \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix} \begin{bmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \\ \vdots \\ \mathbf{l}_n \end{bmatrix} \Delta \mathbf{V} \\ &= \begin{bmatrix} r_{1,1} & r_{2,1} & \cdots & r_{n,1} \\ r_{1,2} & r_{2,2} & & r_{n,2} \\ \vdots & & \ddots & \vdots \\ r_{1,n} & r_{2,n} & \cdots & r_{n,n} \end{bmatrix} \begin{bmatrix} \lambda_1 (\mathbf{l}_1 \cdot \Delta \mathbf{V}) \\ \lambda_2 (\mathbf{l}_2 \cdot \Delta \mathbf{V}) \\ \vdots \\ \lambda_n (\mathbf{l}_n \cdot \Delta \mathbf{V}) \end{bmatrix} \end{aligned} \tag{5}$$

The effect of this transform can be made more clear by investigating how changing voltage affects the change in injected reactive power of a single bus ($\triangle Q_1$ for example). This is shown in (6).

$$\begin{aligned} \triangle Q_1 \quad = \quad & r_{1,1} \lambda_1 (\mathbf{l}_1 \cdot \Delta \mathbf{V}) + r_{2,1} \lambda_2 (\mathbf{l}_2 \cdot \Delta \mathbf{V}) + \\ & \cdots + r_{n,1} \lambda_n (\mathbf{l}_n \cdot \Delta \mathbf{V}) \end{aligned} \tag{6}$$

In order to determine how the reactive power at bus i is affected by the voltage at only bus i, we simply hold all other voltage magnitudes constant. If we choose $i = 1$, the voltage differential vector becomes $\Delta \mathbf{V} = [\ \triangle V_1 \quad 0 \quad \cdots \quad 0\ ]$. The reactive power differential equation changes accordingly.

$$\triangle Q_1 = (\lambda_1 r_{1,1} l_{1,1} + \lambda_2 r_{2,1} l_{2,1} + \cdots + \lambda_n r_{n,1} l_{n,1}) \triangle V_1 \tag{7}$$

At this point, we can define and incorporate the participation factors. The indices in the following equation refer to the j^{th} row and the i^{th} column of the right eigenvector matrix R and the i^{th} row and the j^{th} column of the left eigenvector matrix L.

$$\rho_{i,j} = R_{j,i} L_{i,j} \tag{8}$$

Therefore, $\rho_{i,j}$ defines how the j^{th} state is affected by the i^{th} eigenvalue. Clearly, individual reactive power states can be expressed as a superposition of eigenvalues of varying degrees of participation. If we compute the reactive power changes at each bus based on the voltage changes at each corresponding local bus, we obtain the following set of equations.

$$\triangle Q_1 = (\lambda_1 \rho_{1,1} + \lambda_2 \rho_{2,1} + \cdots + \lambda_n \rho_{n,1}) \triangle V_1$$
$$\triangle Q_2 = (\lambda_1 \rho_{1,2} + \lambda_2 \rho_{2,2} + \cdots + \lambda_n \rho_{n,2}) \triangle V_2$$
$$\vdots$$
$$\triangle Q_n = (\lambda_1 \rho_{1,n} + \lambda_2 \rho_{2,n} + \cdots + \lambda_n \rho_{n,n}) \triangle V_n$$

In these equations, a reactive power state is expressed as a superposition of eigenvalues. Conversely, we can also express each eigenvalue as a superposition of different state contributions. The reason why such an expression is useful is shown through (12). Recognizing that $R = L^{-1}$, the following manipulations may be made.

$$\Delta\mathbf{Q} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \cdots & \mathbf{r}_n \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix} \begin{bmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \\ \vdots \\ \mathbf{l}_n \end{bmatrix} \Delta\mathbf{V}$$

$$\begin{bmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \\ \vdots \\ \mathbf{l}_n \end{bmatrix} \Delta\mathbf{Q} = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_n \end{bmatrix} \begin{bmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \\ \vdots \\ \mathbf{l}_n \end{bmatrix} \Delta\mathbf{V}$$

Now, we can isolate a single eigenvalue (λ_1, for example).

$$\mathbf{l}_1 \Delta\mathbf{Q} = \lambda_1 \mathbf{l}_1 \Delta\mathbf{V} \tag{9}$$

$$\begin{aligned} &l_{1,1}\triangle Q_1 + l_{1,2}\triangle Q_2 + \cdots + l_{1,n}\triangle Q_n = \\ &\lambda_1 \left(l_{1,1}\triangle \mathrm{V}_1 + l_{1,2}\triangle \mathrm{V}_2 + \cdots + l_{1,n}\triangle \mathrm{V}_n\right) \end{aligned} \tag{10}$$

Clearly, the relationship between $\triangle Q$ and $\triangle \mathrm{V}$ for the j^{th} isolated state (holding all else constant) is given by the following expression.

$$\frac{q_{1,j}\triangle \mathrm{V}_j}{q_{1,j}\triangle Q_j} = \frac{\triangle \mathrm{V}_j}{\triangle Q_j} = \frac{1}{\lambda_j} \tag{11}$$

This is true for all states of a given eigenvalue. Therefore, the spectral component that will have the largest voltage variation for a given reactive power change will have the smallest eigenvalue. For this reason, the participation factors of this eigenvalue will be of great interest to study. The j^{th} eigenvalue can be written as a summation of n unique states. In this way, (12) shows how each state participates in the j^{th} eigenvalue of a system.

$$\lambda_j = \lambda_j \rho_{j,1} + \lambda_j \rho_{j,2} + \cdots + \lambda_j \rho_{j,n} \tag{12}$$

There are many different ways to use the eigenvalues and eigenvectors of J_R. For instance, [12] suggest using the smallest eigenvalue of J_R to gauge proximity to bifurcation. Such stability analysis, though, is based solely on the decomposition of a model based static matrix and is highly limited in nature, as outlined by Pal in the discussion section of [12]. Instead, we propose that J_R can be leveraged as tool to combine and thus interpret streams of PMU data. Detecting Critical Slowing Down in time series data is a purely data driven stability assessment, but it can be difficult to understand which nodes will show the strongest EWSs [4]. Therefore, the novel approach outlined in this paper uses results from the static decomposition above to weight and interpret incoming dynamic data.

III. Experimental results: Using participation factors to combine PMU data

A. Polish Test Case Overview

In order to test our methods, we use data from the 2383-bus dynamic Polish test system. This network contains 327 four-variable synchronous generators. Each generator is equipped with a three-variable turbine governor model for frequency control and a four-variable exciter model (AVR) for voltage regulation. There are 322 shunt loads (all connected to generator buses) and 1503 active and reactive loads spread throughout the system. In order to push the system towards voltage collapse, we employed a simple uniform loading of all loads (except for those attached to generator buses). This method is justified in [3]. Half of the PQ bus loads are modeled as voltage controlled loads, while the other half are modeled as frequency controlled loads. Parameters controlling the voltage controlled loads are modeled after the Nordic Test System in [15], while parameters controlling the frequency controlled loads are modeled after the 39 bus test system described in [4].

As in [4], we model this larger network using a set of stochastically forced differential algebraic equations, which can be written as:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}) \tag{13}$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y}, \mathbf{u}) \tag{14}$$

where $\mathbf{f}$ and $\mathbf{g}$ represent the differential and algebraic equations governing the system, $\mathbf{x}$ and $\mathbf{y}$ are the differential and algebraic variables of these equations, and $\mathbf{u}$ represents stochastic power (load or supply) fluctuations. $\mathbf{u}$ follows a mean-reverting Ornstein-Uhlenbeck process:

$$\dot{\mathbf{u}} = -E\mathbf{u} + \boldsymbol{\xi} \tag{15}$$

where E is a diagonal matrix whose diagonal entries equal the inverse correlation times t_{corr}^{-1} of load fluctuations and $\boldsymbol{\xi}$ is a vector of zero-mean independent Gaussian random variables. A further description of our noise model can be found in Sec. II A of [4]. Also given in [4] is a method for analytically computing the covariance and correlation matrices for all state and algebraic variables. We used this method to pre-compute the variance of voltages in the 2383-bus Polish system. After thorough testing, we found the analytically-calculated covariance and correlation matrices to be just as accurate on the large Polish system as they were on the small 39 bus system. Thus, the data presented in the following two sections use the analytically calculated results rather than averaged dynamic simulation results.

In order to push the system towards a critical transition (voltage collapse), we increase all loads and generator set points by a constant loading factor b, which ranges from $b = 1$ up to $b = 1.92$. We empirically found that voltage collapse occurs when the load factor increases past $b = 1.923$.

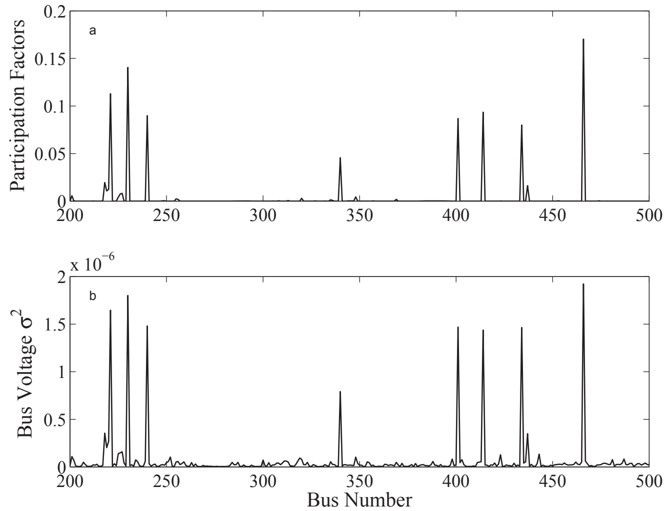

Figure 1. Participation factors and voltage variance values for buses 200 through 500 from the loaded 2383 bus system. Bus voltage variances (below) correlate almost perfectly with the participation factors of the smallest eigenvalue of the RPFJ (above).

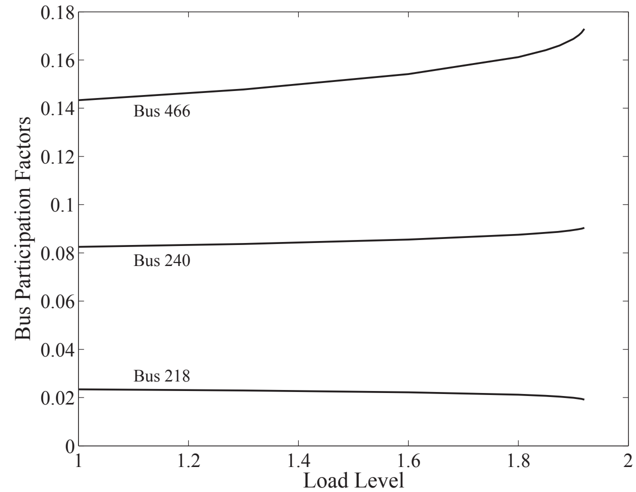

Figure 2. Participation factors for three different buses at different load levels. As the system is increasingly loaded (right up to bifurcation), bus 466 (the most unstable bus) sees a sharp increase in participation to the instability. Bus 240 (the 5th most unstable bus) sees a very slight increase, while bus 218 (the 10th most unstable bus) begins to see a decrease.

The concept of a limit-induced bifurcation is an important topic discussed in [16]. Power system limits, such as reactive power generation limits, are an important aspect of stability analysis. However, in order to focus our analysis on voltage collapse without the possibility of additional bifurcations due to limits, we increased limits in our test case so that the system can run up to $b = 1.92$ without hitting a limit-induced bifurcation. This simplification allows us to focus our study on the effects of pure voltage collapse. Future work will extend our method to the case of other types of bifurcations.

B. Evidence for Bus Voltage Variance Prediction

As indicated by (11), the smallest eigenvalue of J_R corresponds to the spectral component that will yield the largest voltage variation for a given variation in reactive power. When the participation factors corresponding to the smallest eigenvalue are plotted, they are shown to directly predict the relative bus voltage variance strengths. Fig. 1 shows two plots. The top plot corresponds to the participation factors for buses 200 through 500, and the bottom plot shows the true bus voltage variance, derived analytically, for buses 200 through 500. The remaining system buses are left out for the sake of clarity. Despite the fact that the participation factors are completely blind to the dynamics of the system, they are still quite successful at predicting the relative variance strengths.

As shown in reference [17], increasing voltage variance is due to buses that are operating closer to the limit along the PV curve. Therefore, participation factors of the smallest eigenvalue also identify the node voltages which, as the system is overloaded, begin to diverge from their nominal values. These tend to be the nodes that are primarily responsible for non-convergence in the power flow equations. Interestingly, as PQ buses in the system are increasingly loaded, the recalculated participation factors do not change drastically (for a uniform loading condition). This is equivalent to saying that the *spectral components* do not change significantly. This is a useful result, since state-estimator derived power flow models are only typically computed periodically during power systems operations.

As indicted previously, participation factors of the most unstable nodes provide a very clear indication of the relative bus voltage variance strengths. Therefore, as the system is increasingly loaded, the most unstable nodes will begin to have larger and larger participation factors as their relative variance strengths grow relative to other, more stable nodes. Fig. 2 shows an example of this for the 2383 bus system. As the system is loaded, the relative strength of the most unstable bus' participation increases almost linearly, but when the critical transition approaches, the participation begins to climb more steeply.

C. Evidence for Locating Generators with Elevated Voltage Autocorrelation

CSD theory predicts that signals from a system approaching a critical transition will show increasing levels of autocorrelation, $R(\Delta t)$. This can be due to the system's reduced ability to respond to high frequency fluctuations [18], but the system also begins to return to an equilibrium state more slowly after perturbations [17]. In a power system, system-wide increases inautocorrelation are typically indicators of increasingly unstable generator dynamics. These dynamics are driven by the load variations, since this is where the noise is being injected.

Fig. 1, clearly shows that a small number of buses in our test case have particularly high voltage variances measurements. Looking at the topology of this network, we find that these nodes are in fact separated by only a small number of transmission lines suggesting that these buses represent a weak load pocket. The participation factors are thus useful for

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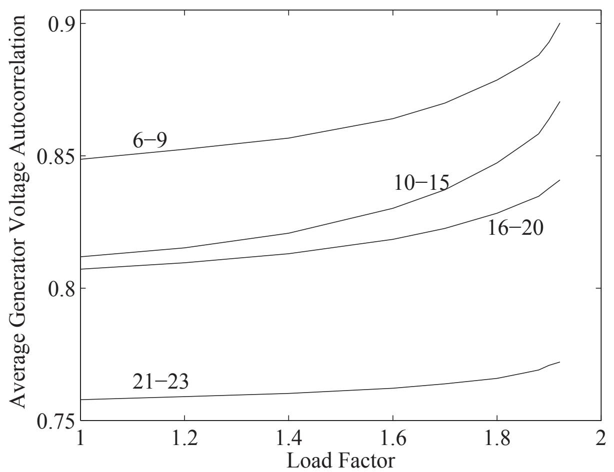

Figure 3. Average autocorrelation of voltage magnitude measurements for several different groups of generators at varying load levels. The top trace is all generators located between 6 and 9 transmission lines (hops) from the load pocket. The second groupincludes generators 10 and 15 transmission lines distant from the load pocket. The third group includes generators 16 and 20 lines from the pocket, and the fourth group is generators between 21 and 23 lines distant.

identifying load pockets. Many of the buses connected to this pocket show high variance, and are therefore driving the autocorrelation of the most proximal generators. By identifying generator proximity to unstable loads, the autocorrelation of the output signals (voltage and current) of close generators can be scrutinized.

To study this further, the 327 generators of the Polish system were group according to their distance (quantified by line count) from the load pocket center. Next, the system wide algebraic correlation matrix was derived for a series of increasing load parameters. For each generator grouping, the average bus voltage autocorrelation value was computed and plotted. Fig. 3 shows the average voltage autocorrelation, with a time lag of $\triangle t = .2$s, for several different group of generators and varying load levels. (See [17] for a discussion of this choice for Δt.).The generators are grouped (and identified by) their proximity to the load pocket shown in Fig. 1. Clearly, the generators that are closest to this load pocket show the largest average autocorrelation statistics. This is a very useful result.

IV. Conclusions

This paper presents evidence that participation factors from a spectral decomposition of the reduced power flow Jacobian can be used to design methods for combining sychrophasor measurements to produce system-wide indicators of instability in power systems. Our approach uses spectral information from the power flow Jacobian, which can be updated every few minutes through the SCADA network in combination with high sample-rate voltage magnitude measurements, which can be collected from synchronized phasor measurement systems deployed throughout the system. The results suggest that that a combination of a power flow model and streaming PMU data analysis can be used to be used to develop system wide stability metrics. The detailed development of these metrics is a topic for future research.

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