Decentralized stability rules for microgrids

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Abstract—**Stability certification of microgrids can be challenging due to the lack of information on exact values of system parameters. Moreover, full-scale direct stability analysis for every system configuration can be economically and technically unjustified. There exist a demand for simple conditions imposed on system components that guarantee the whole system stability under arbitrary interconnections. Most of existing methods are relying on the so-called passivity property which can be difficult to realize by all the system components simultaneously. In the present manuscript we develop an approach for certifying the system stability by separately considering its properties in different regions of frequency domain. We illustrate our method on the case of droop-controlled inverters and show that while these inverters can never be made passive, reasonable stability certificates can be formulated by careful consideration of their input admittance for different frequency regions. We discuss the generalization of the method for different types of microgrid components.**

Index Terms—**Small-signal stability, microgrids, dissipativity, plug-and-play.**

I. Introduction

Microgrids are becoming an increasingly popular topic for both academic and industrial society [1]. The advances in power electronics technologies have lead to significant decrease in renewable generation costs which inspired discussions about splitting the existing distribution grids into autonomous systems. Subsequently, there have been a significant progress in development of control architectures for power electronics-interfaced generation allowing for flexible microgrid operation [2], [3]. It was soon realized that control methods that were standard for large scale power systems have rather limited applications in microgirds due to stability constraints [4]. Moreover, modelling approaches (ex. modelling based on time-scale separation) routinely used for conventional power systems appeared to be inadequate for microgrids, which demands for new modeling techniques to be developed [5].

It is always possible to refer to full-scale dynamic models for stability analysis of microgrids, directly calculating the eigenvalues of the state matrix for specific operating point. However, such an approach assumes the full knowledge of system configuration which is much less likely for a microgrid than for a conventional power system. Moreover, performing full-scale stability analysis for every possible microgrid configuration is most likely economically and technically unjustified. There is, therefore, the need for simple but reliable stability certificates that can be routinely used for a wide class of microgrid configurations. Ideally one thinks about developing of specific standards for typical microgrid components that will allow stable operation under arbitrary interconnection.

In our recent work [6] we have developed a low-dimensional model for inverter-based microgrids which allowed us to uncover the main sources of instabilities and paved the way towards development of completely decentralized interconnection rules for such systems [7]. However, our methods were still reliant on rather specific dynamic models of system components (namely, droop-controlled inverters) and assumed at least partial knowledge about the system configuration. As any other method based on certain dynamic models, it requires specific extension in order to include new types of components (ex. current source inverters, synchronous machines etc.) and assumes the knowledge of their control settings. Moreover, stability certificates formulated in [7], while being decentralized, depend not only on the settings of the system components, but also on their interconnection.

The celebrated concept of *dissipative* dynamic systems [8], [9] allows for formulation of stability certificates for the whole system through the separate consideration of its components: if every component of the system is dissipative, then the whole system is also dissipative, therefore stable, irrespective to the way components are interconnected. A more specific passivity property has allowed the formulation of rather simple (although not always easily realisable) constraints on input admittance/impedance of power system components [10]–[12]. The advantage of such an approach is that input admittances of individual components does not have to be known but can simply be measured. However, it is not straightforward to apply the method to components that are not passive and can not be made so by simple adjustments of their control settings.

In the present manuscript we develop a method for stability certification based on a special type of dissipativity approach that relies on different certificates in different regions of frequency domain. It is especially applicable for systems that are not naturally passive (and can not be made so by simple adjustment of their control settings). We illustrate our method on the example of inverter-based microgrids, and discuss how it can be generalized to arbitrary systems.

II. Admittance Representation

In this section we describe the concept of small-signal effective admittance that are then used to develop stability certificates. Let us consider a 3-phase AC power system (single-phase systems can be treated in a similar way) with arbitrary types of loads and generators and assume that the system has some equilibrium operating frequency Ω_0.

Let us start from representing bus voltages and line currents as 3-component vectors, for example voltage at any bus:

$$\mathcal{V}(t) = [\mathcal{V}_a(t), \mathcal{V}_b(t), \mathcal{V}_c(t)]^T \quad (1)$$

where we explicitly indicate t as an argument to emphasize that we are still in time domain. In order to perform stability analysis it is convenient to first switch to the so-called $d-q$ representation in a synchronous reference frame, rotating with equilibrium frequency Ω_0:

$$V(t) = [V_d(t), V_q(t)]^T = \mathbf{T}\mathcal{V}_{abc}(t)e^{-j\Omega_0 t} \quad (2)$$

where transformation matrix $\mathbf{T}$ is [11]:

$$\mathbf{T} = \sqrt{\frac{3}{2}} \begin{bmatrix} \frac{2}{3} & 0 \\ -\frac{1}{3} & \frac{1}{\sqrt{3}} \\ -\frac{1}{3} & -\frac{1}{\sqrt{3}} \end{bmatrix}^T \quad (3)$$

Line currents are transformed in similar way. The convenience of such transformation is that at equilibrium all the $d-q$ voltage and current vectors are constant. If all the power system components are *symmetric* then $d-$ and $q-$ components can be represented as real and imaginary parts of the corresponding complex variable [11]. In this manuscript we will be dealing with both *symmetric* and *unsymmetric* components, and therefore, we will follow the vector representation.

We further introduce the small-signal variations of voltage and current:

$$V_i(t) = V_i^0 + v_i(t); \qquad I_{ik}(t) = I_{ik}^0 + i_{ik}(t) \quad (4)$$

where the superscript 0 marks the equilibrium values and subscript i refers to bus number while subscript ik refers to line between buses i and k. In the rest of the manuscript we will refer to small-signal perturbations simply as voltage and current. Both $v_i(t)$ and $i_{ik}(t)$ are 2-dimensional vectors obtained similarly to (2).

Let $v = [v_1, v_2, ...v_N]^T$ be the $2N$ dimensional vector (N - is the total number of buses in the system) of bus voltages and $i_L = [i_{L1}, i_{L2}, ...i_{LN}]^T$ be the vector of load currents, we assume the positive direction for this current to be from the node to the ground (through the load). Similarly to conventional phasor representation, bus voltages and load currents (generators are also included in the "load" set) in Laplace domain are related by the network admittance matrix $\mathbf{Y}_N$:

$$i_L = -\mathbf{Y}_N(s)v \quad (5)$$

Since both voltages and line currents are 2-dimensional vectors, admittances of individual lines $\mathbf{y}_{ik}$ are 2×2 matrices, rather than just complex numbers. Throughout the rest of the paper we will use bold lower-case letters to denote 2×2 matrices. The network admittance matrix $\mathbf{Y}_N$ is composed from these blocks (explicit expressions for blocks $\mathbf{y}_{ik}$ are presented in Sec. IV-A) according to conventional rules: non-diagonal blocks are the negative admittances of corresponding lines (or zero if there is no line between corresponding buses), and the diagonal blocks are the sums of the admittances of all the lines connected to the bus [13]:

$$\mathbf{Y}_N = \begin{bmatrix} \sum\limits_k \mathbf{y}_{ik} & \dots & -\mathbf{y}_{ik} \\ \vdots & \ddots & \vdots \\ -\mathbf{y}_{ik} & \dots & \sum\limits_i \mathbf{y}_{ik} \end{bmatrix} \quad (6)$$

Thus $\mathbf{Y}_N$ is a $2N \times 2N$ matrix.

On the other hand, load current vector can be written in terms of the load admittance matrix $\mathbf{Y}_L$ which has load input admittances as diagonal blocks and zeros elsewhere: $\mathbf{Y}_L = \text{diag}(\mathbf{y}_1 \dots \mathbf{y}_N)$. Then the relation between voltage and load current is:

$$i_L = \mathbf{Y}_L(s)v \quad (7)$$

Let us also denote simply as $\mathbf{Y}$ the sum of network and load admittance matrices. Then, combining (5) and (7) we have:

$$\mathbf{Y}(s)v = [\mathbf{Y}_N(s) + \mathbf{Y}_L(s)]\, v = 0 \quad (8)$$

that should be satisfied for any bus voltage vector v in the absence of external disturbances to the grid.

III. Stability Certificates

In this section we formulate stability certificates using the system admittance matrix $\mathbf{Y}$ and show how these certificates can be made completely decentralized. Our method is along the lines with the celebrated *dissipativity* concept, however, we formulate it here from the basic principles in order to make the approach more transparent. Detailed discussion of equivalence of this method to the method of dissipative systems is the subject of subsequent publications.

A. General Formulation in Terms of Admittance Matrix

We start by noticing that every voltage vector v satisfying (8) corresponds to one eigenmode of the system. Since equation (8) has nontrivial solutions only for values of $s = s_0$ for which the determinant of admittance matrix is zero, the time-domain dynamics of any eigenmode can be written as:

$$v(t) = v(0)e^{s_0 t} \quad (9)$$

This leads us to the following theorem:

Theorem 1. *The system described by the admittance matrix* $\mathbf{Y}$ *is stable if and only if all the roots* s_0 *of the equation*

$$\det[\mathbf{Y}(s)] = 0 \quad (10)$$

lie in the left-hand side of the complex plane.

Proof. The system is small-signal stable if all of its eigenmodes decay with time in the absence of external perturbations. According to equation (9) the decay rate of any eigenmode is determined by the real part of s_0 which is one of the solutions of equation (10). Therefore, if $\mathbf{Re}[s_0] < 0$ for all s_0, the system is stable. □

While Theorem 1 gives the necessary and sufficient condition for stability of the system its use is equivalent to direct analysis of the system state matrix and does not bring any practical advantage. In order to derive practically convenient stability conditions, we first notice, that if the system settings change from a stable to an unstable point, one or more solutions of the equation (10) cross the imaginary axis. Let us now introduce some parameter α that parametrizes the system settings with $\alpha = 0$ corresponding to a definitely stable set up and $\alpha = 1$ to the target set up for which stability has to be certified. The following theorem allows one to certify stability of the system by screening the determinant of the admittance matrix over real values of frequency, corresponding to $s = j\omega$, rather than finding complex roots s_0.

Theorem 2. *The system described by the admittance matrix* $\mathbf{Y}$ *is stable if and only if:*

$$\det\left[\mathbf{Y}(\omega, \alpha)\right] \neq 0 \tag{11}$$

for all real ω *and all* $\alpha \in [0, 1]$.

Proof. Since the system is stable for $\alpha = 0$ all the roots of equation (10) are in the left hand plane. If condition (11) is satisfied while α increases from 0 to 1 then none of the roots have moved to the right plane, so the system remains stable. □

Remark 1. *Condition* (11) *remains valid if* $\mathbf{Y}$ *is multiplied by arbitrary non-singular matrices, which in general can be functions of frequency and* α. *Thus, we can write the generalization of coondition* (11) *as:*

$$\det\left[\mathbf{M}\mathbf{Y}(\omega, \alpha)\mathbf{\Gamma}\right] \neq 0 \tag{12}$$

where $\mathbf{M}(\omega,\, \alpha)$ *and* $\mathbf{\Gamma}(\omega,\, \alpha)$ *are arbitrary non-singular matrices.*

Consider now a system which is composed from two or more subsystems for every one of which stability is certified. The admittance matrix of the whole system can always be split into the sum of matrices each related to one subsystem. However, the use of either (10) or (12) is not straightforward since there is no simple way to relate the determinant of the full admittance matrix to determinants of its submatrices corresponding to individual subsystems. However, instead of demanding admittance matrix being non-singular, one can demand it's Hermitian part to be positive definite. This condition, known as *passivity*, possesses the "addittiveness" property: if all the components of a system are passive, than the whole system is also passive, therefore - stable. However, equation (12) allows to formulate an even more general sufficient stability condition:

Theorem 3. *If there exist two matrices* $\mathbf{M}(\omega,\, \alpha)$ *and* $\mathbf{\Gamma}(\omega,\, \alpha)$ *such that for any real* ω *and all values of* $\alpha \in [0, 1]$ *the following conditions are satisfied:*

$$\det\left[\mathbf{M}(\omega,\, \alpha)\right] \neq 0; \qquad \det\left[\mathbf{\Gamma}(\omega,\, \alpha)\right] \neq 0 \tag{13}$$

$$\left[(\mathbf{M}\mathbf{Y}\mathbf{\Gamma}) + (\mathbf{M}\mathbf{Y}\mathbf{\Gamma})^{\dagger}\right] \succ 0 \tag{14}$$

then the system is stable.

Proof. Suppose that $\mathbf{Y}$ is singular for some values of ω and α. Then $\mathbf{M}\mathbf{Y}\mathbf{\Gamma}$ is also singular for any choice of $\mathbf{M}$ and $\mathbf{\Gamma}$ (as long as they are both non-singular), so the matrix in the left of (14) has at least on zero eigenvalue, therefore, can not be strictly positive definite whatever the choice of $\mathbf{M}$ and $\mathbf{\Gamma}$ is. □

Theorem 3 can be interpreted both ways. Thus if the system is stable, there necessarily exist some matrices $\mathbf{M}$ and $\mathbf{\Gamma}$ (in fact families of such matrices) that satisfy (13) and (14) (apart from). To see this, one can consider a simple choice: $\mathbf{M} = \mathbf{Y}^{\dagger}$ and $\mathbf{\Gamma} = \mathbf{I}$, so that (14) becomes:

$$2\mathbf{Y}^{\dagger}\mathbf{Y} \succ 0 \tag{15}$$

which is equivalent to (11).

B. Decentralized Stability Certificates

The key to formulating decentralizaed stability conditions lie in the fact that the system admittance matrix can be easily split into a sum of matrices, with each one responsible for either single load or single line (we will refer to them as component matrices):

$$\mathbf{Y} = \sum_{i,k} \mathbf{Y}_N^{ik} + \sum_{i} \mathbf{Y}_L^{i} \tag{16}$$

Here $\mathbf{Y}_N^{ik}$ and $\mathbf{Y}_L^{i}$ are the matrices with non-zero element corresponding only to line ik and load i respectively. Each network component matrix $\mathbf{Y}_N^{ik}$ has only four non-zero blocks (see (6)) - two diagonal and two off diagonal, while each load component matrix $\mathbf{Y}_L^{i}$ - only one diagonal non-zero block.

If one now chooses the factors $\mathbf{M}$ and $\mathbf{\Gamma}$ in such a way that the property (16) still holds for the resulting $\mathbf{M}\mathbf{Y}\mathbf{\Gamma}$ matrix - one can apply condition (14) separately to every term in (16), i.e. to every component of the system. One of such choices for $\mathbf{M}$ and $\mathbf{\Gamma}$ are block diagonal matrices (composed of 2×2 blocks). In the present manuscript we will analyze the case when $\mathbf{M} = \mathrm{diag}(\mathbf{m}, \ldots \mathbf{m})$ - all diagonal blocks are equal to the same 2×2 matrix $\mathbf{m}$, and $\mathbf{\Gamma} = \mathbf{I}$. In this case the resulting component matrices for each line will be composed of blocks $\mathbf{m}\mathbf{y}_{ik}$:

$$\mathbf{M}\mathbf{Y}_N^{ik} = \begin{bmatrix} \mathbf{m}\mathbf{y}_{ik} & -\mathbf{m}\mathbf{y}_{ik} & \ldots \\ -\mathbf{m}\mathbf{y}_{ik} & \mathbf{m}\mathbf{y}_{ik} & \ldots \\ \ldots & \ldots & \ldots \end{bmatrix} \tag{17}$$

where we have moved all the non-zero blocks to the upper-left corner. Two eigenvalues of this matrix a doubled eigenvalues of matrix $\mathbf{m}\mathbf{y}_{ik}$ and all the rest are zero. Therefore, this matrix is positive semi-definite whenever the matrix $\mathbf{m}\mathbf{y}_{ik}$ is positive (semi)-definite. Transformation of the component matrices for loads from (16) is even simpler, their diagonal blocks just become $\mathbf{m}\mathbf{y}_i$.

Remark 2. *Since each component matrices from* (16) *have all elements zero except for those referring to one bus (for load matrices* $\mathbf{Y}_L^{i}$ *), or pair of buses (for line matrices* $\mathbf{Y}_N^{ik}$ *), all of these matrices have zero eigenvalues. However, as long as condition* (14) *is satisfied for* every *load input admittance* *matrix $\mathbf{y}_i$, component matrices $\mathbf{Y}_L^i$ have no common null-vectors, so that* (14) *is satisfied for the full admittance matrix.*

IV. Inverter-Based Microgrids: Admittance Matrix

In this section we will derive explicit expressions for admittance matrix of the components of inverter-based microgrids. We assume that system consists of a number of droop-controlled inverters and a number of constant impedance loads.

A. Line and load admittances

The blocks $\mathbf{y}_{ik}$ from the system admittance matrix corresponding to lines can be directly obtained from time-domain Kirschoff's equations for small-signal variations of current and voltage. For the line current between buses i and k we have (we omit the subscript ik for current and line parameters to simplify denotations):

$$L\frac{di_{\mathrm{d}}}{dt} = v_{\mathrm{d},i} - v_{\mathrm{d},k} - Ri_{\mathrm{d}} + \omega_0 Li_{\mathrm{q}} \tag{18a}$$

$$L\frac{di_{\mathrm{q}}}{dt} = v_{\mathrm{q},i} - v_{\mathrm{q},k} - Ri_{\mathrm{q}} - \omega_0 Li_{\mathrm{d}} \tag{18b}$$

In the frequency domain we have:

$$i(\omega) = \mathbf{y}_{ik}(\omega)\left[v_i(\omega) - v_k(\omega)\right] \tag{19}$$

with the following expression for $\mathbf{y}_{ik}$:

$$\mathbf{y}_{ik} = \begin{bmatrix} R + j\omega L & -X \\ X & R + j\omega L \end{bmatrix}^{-1} \tag{20}$$

where $X = \omega_0 L$. This expression is also valid for the admittance matrix of a passive load.

B. Inverter input admittance

We assume that inverter terminals are connected to the system buses via certain impedances which, in general, is the combined virtual impedance, coupling impedance, and possibly connecting line. We will refer to this combined impedance simply as coupling impedance and denote it as z_c. Let E, Θ, and Ω denote the magnitude, angle, and frequency of inverter terminal voltage. The following equations describe the time-domain dynamics of a droop-controlled inverter [6] (we omit the subscript i denoting the inverter number):

$$\frac{d\Theta}{dt} = \Omega - \Omega_0 \tag{21a}$$

$$\tau\frac{d\Omega}{dt} = \Omega^* - \Omega - n_p P \tag{21b}$$

$$\tau\frac{dE}{dt} = E^* - E - n_q Q \tag{21c}$$

here P and Q are instantaneous values of real and reactive power discharged by the inverter, and k_p and k_q are frequency and voltage droop coefficients respectively. Constants Ω^* and E^* are the frequency and voltage set-points respectively and τ_i is the inverse of the power controller filter cut-off frequency w_{ci} (it is typically around 31.41 rad/s for 50Hz grids).

As previously, let the lower-case letters denote small-signal variations of corresponding variable. Thus θ, ω, e, p, and q are the variations of inverter terminal angle, frequency, voltage magnitude, real and reactive power output respectively. In writing the small-signal representation of equations (21) we take into account that inverters typically operate at very low values of terminal angle and voltage variation due to small per unit values of coupling impedance (for details see [6]). In this case we have the following simple relations (we assume that the nominal voltage is 1 pu):

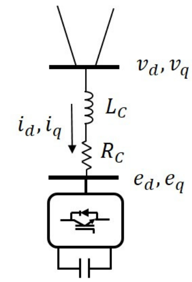

Fig. 1. Inverter connected to a grid node: v_d and v_q are small-signal variations of grid node voltage while e_d and e_q are variations of inverter terminal voltage. The combined coupling and virtual inductance and resistance are L_c and R_c respectively.

$$e_q = \theta; \qquad e_d = e \tag{22a}$$

$$p = i_d; \qquad q = -i_q \tag{22b}$$

Where e_d, e_q, i_q, and i_q are d and q components (in small-signal sense) of inverter terminal voltage and inverter current (Fig. 1). Then the small-signal approximation of equations (21) becomes:

$$\tau\frac{d^2 e_q}{dt^2} + \frac{de_q}{dt} = k_p i_d \tag{23a}$$

$$\tau\frac{de_d}{dt} + e_d = -k_q i_q \tag{23b}$$

From this we get the effective admittance matrix of inverter controls $\mathbf{y}_{ctrl}$ that links terminal voltage and inverter current $[i_d, i_q]^T = \mathbf{y}_{ctrl}[e_d, e_q]^T$. The explicit expression for $\mathbf{y}_{ctrl}$ is:

$$\mathbf{y}_{ctrl} = \begin{bmatrix} 0 & -\frac{k_q}{1+j\omega} \\ -\frac{k_p}{\omega^2 - j\omega} & 0 \end{bmatrix}^{-1} \tag{24}$$

In order to get the full inverter input admittance we note that the coupling impedance is connected in series with the inverter and has the admittance given by (20). Therefore, for inverter i input admittance we have:

$$\mathbf{y}_i = \begin{bmatrix} R_c + j\omega L_c & -X_c - \frac{k_q}{1+j\omega} \\ X_c - \frac{k_p}{\omega^2 - j\omega} & R_c + j\omega L_c \end{bmatrix}^{-1} \tag{25}$$

where the subscript c means that the corresponding parameter refers to coupling impedance.

V. Stability of Inverter-Based Microgrids

We are now in the position to construct fully decentralized stability certificates that are valid for inverter-based microgrids of arbitrary size. First, we introduce to our system parameter α according to (11). A simple choice is to multiply both k_p

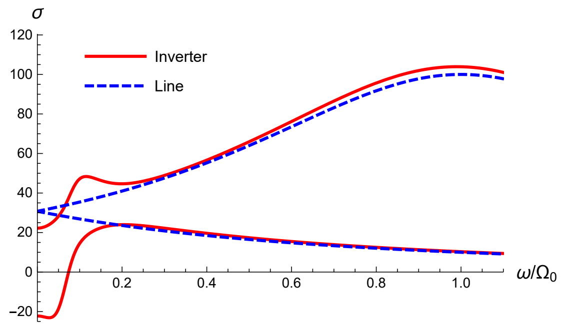

Fig. 2. Eigenvalues of $(\mathbf{y}+\mathbf{y}^{\dagger})/2$ for line admittance (20) (dashed blue) and inverter admittance (25) (red).

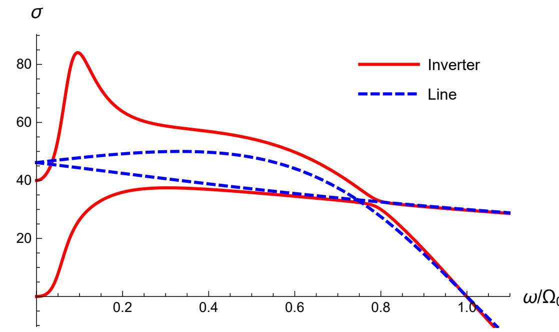

Fig. 3. Eigenvalues of $[(\mathbf{m}\mathbf{y})+(\mathbf{m}\mathbf{y})^{\dagger}]/2$ with $\mathbf{m}$ given by (26) and line admittance from (20) (dashed blue) and inverter admittance from (25) (red).

and k_q coefficients in (25) by α, so that $\alpha = 0$ corresponds to a definitely stable system.

As the base case we use the system with the following parameters. Total coupling resistance and inductance of inverter are $R_c = 0.01$ pu and $X_c = 0.015$ pu respectively (per unit values of impedances in inverter-based microgrids are always small [6]). Inverter frequency droop coefficient $k_p = 1.3 \cdot 10^{-3}$ pu (0.13%) and voltage droop coefficient $k_q = 7.5 \cdot 10^{-3}$ pu (0.75%). Network lines inductance and resistance values are chosen the same as corresponding inverter coupling inductance and resistance, however, particular values of network parameters do not influence stability certificates that we present in this section.

We start by choosing $\mathbf{m} = \mathbf{I}$ which corresponds to conventional passivity condition. Fig. 2 shows the eigenvalues σ of $(\mathbf{y}+\mathbf{y}^{\dagger})/2$ as functions of frequency for inverter admittance matrix from (25), and line admittance matrix from (20). At frequencies $\omega > \Omega_0$ eigenvalues of both matrices remain positive[1] for any values of coupling impedance and inverter droop coefficients. We notice, that eigenvalues corresponding to line admittance matrix as well as to constant impedance load are always positive (irrespective of values of inductance and resistance), which means that these components are *passive*. Droop-controlled inverter, however, is not passive and can not be made passive by adding more virtual/coupling impedance or by reducing droop coefficients - standard ways for enhancing stability of such inverters. One of the eigenvalues corresponding to matrix (25) always stays negative for low values of ω.

From Fig.2 we notice, that eigenvalues corresponding to line admittance matrix, as well as constant impedance load admittance matrix are always positive, which means that these components are *passive*. Droop-controlled inverter, however, is not passive and can not be made passive by adding more virtual/coupling impedance or by reducing droop coefficients - standard ways for enhancing stability of such inverters. One of the eigenvalues corresponding to matrix (25) always stays negative for low values of ω.

[1]Eigenvalues in Fig.2 approach zero as $\omega \to +\infty$. This issue can be resolved by placing a small shunt capacitor in parallel with inverter (which makes both eigenvalues to stay positive), since all real networks always have shunt capacitance present naturally.

Remark 3. *Another example of a system that can not be made passive is DC constant power load (CPL) realized by a power electronics converter. A practical way to stabilize the grid with such loads is to place capacitors to load buses. However, this will not make the CPL's passive: irrespective of the capacitor size the real part of the combined input admittance of load and capacitor will still be negative for sufficiently small frequencies [14].*

In order to certify stability of the inverter from (25) we now need to find some matrix $\mathbf{m}$ that will make both eigenvalues of $(\mathbf{m}\mathbf{y}_i)+(\mathbf{m}\mathbf{y}_i)^{\dagger}$ positive. However, it is sufficient to do this only for small frequencies, where the choice of $\mathbf{m} = \mathbf{I}$ can not provide the needed certificate. Fig. 3 shows the eigenvalues [2] for the system with the following choice of $\mathbf{m}$:

$$\mathbf{m} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{26}$$

One of the eigenvalues corresponding to both inverter and line always becomes negative for $\omega = \Omega_0$ which is irrelevant to us, since in this frequency region stability is already certified by $\mathbf{m} = \mathbf{I}$ (Fig.2).

Thus, with a simple choice of matrix $\mathbf{m}$ for different frequency regions we were able to certify stability of an inverter-based microgrids with parameters specified above. We note, that our certificates can be applied to systems with arbitrary number of inverters and constant impedance loads, with network parameters taking any values while the values of inverter coupling impedance should be fixed. The voltage and frequency droop coefficients should be less or equal to the above specified values: $k_p = 1.3 \cdot 10^{-3}$ pu (0.13%), and $k_q = 7.5 \cdot 10^{-3}$ pu (0.75%). We note that these limits are rather conservative for the chosen values of coupling impedance and a more diligent choice of matrices $\mathbf{M}$ and $\mathbf{\Gamma}$ could provide a better result.

We note, that the above limit on frequency droop coefficient is rather tight from the practical point of view. Let us now illustrate our method on the example of stability enhancement by virtual impedance. It is known, that adding virtual

[2]One of the eigenvalues from Fig.3 for inverter is zero at $\omega = 0$ due to phase shift invariance of the system. This is regularized by introducing a small secondary control term to inverter dynamic equation (21b)

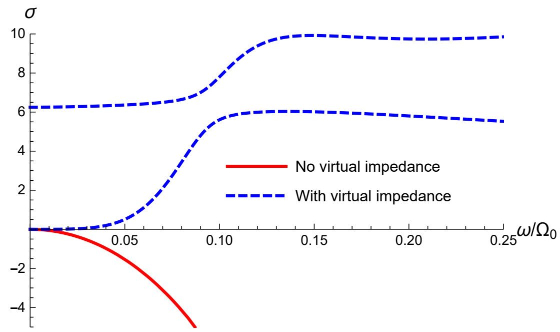

Fig. 4. Eigenvalues of $[(\mathbf{my}) + (\mathbf{my})^{\dagger}]/2$ with $\mathbf{m}$ given by (26) and inverter admittance (25) with (dashed blue) and without (red) additional virtual impedance.

impedance to inverter control loop increases stability region in terms of frequency and voltage droop coefficients [15]. Fig. 4 illustrates the stability certification for inverter with the total value of $X_c = 0.15$ pu of coupling and virtual reactance (and corresponding value of inductance). Such a set up allows one to certify stability for the values of frequency and voltage droop coefficients of $k_p = 1.5{\cdot}10^{-2}$ pu (1.5%), and $k_q = 10^{-2}$ pu (1.0%) - the blue dashed curves in Fig. 4 correspond to eigenvalues of $[(\mathbf{my})+(\mathbf{my})^{\dagger}]/2$ for system with these values of parameters and $\mathbf{m}$ from (26). The red curve shows one of the eigenvalues of the matrix $[(\mathbf{my}) + (\mathbf{my})^{\dagger}]/2$ with the specified values of droop coefficients but without additional virtual impedance, for which stability can not be certified.

VI. Conclusion

We proposed an approach for stability certification of arbitrary power grids by imposing separate conditions on its components. Condition (14) allowed us to step beyond the traditional passivity approach and use different certificates in different regions of frequency domain. Thus, we showed that a conventional droop controlled inverter can never be made passive component, so there is now way to certify its stability by demanding the positive definiteness of the Hermitian part of its input admittance over the whole range of frequencies. On the other hand a rather simple choice of the matrix $\mathbf{M}$ allowed us to obtain reasonable stability certificates by separately considering low and high frequency domains. The method developed in this manuscript can be improved in several ways.

First, the analysis of different typical components for power grids can be done similarly to the way droop-controlled inverters were analyzed. It seems that synchronous machines, current source inverters and induction machines will account for the vast majority of possible components (in addition to constant impedance loads and droop-controlled inverters) of modern power grids. Diligent choice of matrices $\mathbf{M}$ and $\mathbf{\Gamma}$ in (14) can provide certain compromise for simultaneous stability certification of different types of components. It is especially valuable for microgrids applications since their stability certification by direct modelling can be unjustified and formulation of simple, although conservative rules for main components can be much more practical.

Second, several separate regions in frequency domains can be considered (we have only consider two regions in the present manuscript) depending on the components particular input admittance. The intuition behind is that instabilities are typically caused by just few control parameters and are associated with modes in a rather narrow frequency region. One can think about expanding the input admittances of system components in power series around these critical frequencies and obtaining analytic expressions for constraints on different control settings.

Finally, it seems to be promising to consider the presence of non-dissipative components as long as their supply rate can be properly quantified and bounded. As a simple example one can think about the choice of $\mathbf{M}$ and $\mathbf{\Gamma}$ that make the network "active" with subsequent constraints on line parameters. Such an approach can allow other components of the system to be made more dissipative, so that the overall stability will be certified with less conservativeness.

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